

Write and **prove** a formula for $\sinh(x+y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$.

SCORE: _____ / 6 PTS

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y \quad (1)$$

$$= \left[\frac{e^x - e^{-x}}{2} \frac{e^y + e^{-y}}{2} + \frac{e^x + e^{-x}}{2} \frac{e^y - e^{-y}}{2} \right] \quad (1)$$

$$= \frac{(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}) + (e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y})}{4} \quad (1)$$

$$= \frac{2e^{x+y} - 2e^{-x-y}}{4} \quad (1)$$

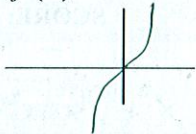
$$= \frac{e^{x+y} - e^{-(x+y)}}{2} \quad (1)$$

Sketch the general shape and position of the following graphs.
(Don't worry about specific x - or y - coordinates.)

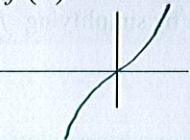
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SCORE: _____ / 3 PTS

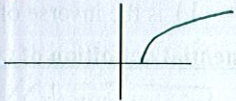
$$f(x) = \tanh^{-1} x$$



$$f(x) = \sinh x$$



$$f(x) = \cosh^{-1} x$$



There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry.

SCORE: _____ / 7 PTS

- [a] Write that identity involving $\sinh x$ and $\cosh x$. You do NOT need to prove the identity.

$$\cosh^2 x - \sinh^2 x = 1, \textcircled{1}$$

- [b] Write the identity for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$ simultaneously. You do NOT need to prove the identity.

$$\cosh 2x = \cosh^2 x + \sinh^2 x, \textcircled{1}$$

- [c] Use the results of [a] and [b] to find and prove an identity for $\cosh 2x$ that uses only $\sinh x$.

$$\begin{aligned} \cosh 2x &= (1 + \sinh^2 x) + \sinh^2 x, \textcircled{1} \\ &= 1 + 2\sinh^2 x, \textcircled{\frac{1}{2}} \end{aligned}$$

- [d] If $\tanh x = -\frac{2}{3}$, find $\sinh x$ using identities.

You must explicitly show the use of the identities but you do NOT need to prove the identities.

Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.

$$\begin{aligned} \operatorname{sech}^2 x &= 1 - \tanh^2 x \\ &= 1 - \frac{4}{9}, \textcircled{1} \\ &= \frac{5}{9} \end{aligned}$$

$$\operatorname{sech} x = \frac{\sqrt{5}}{3}, \textcircled{\frac{1}{2}}$$

SINCE $\operatorname{sech} x > 0$ FOR ALL $x \in \mathbb{R}$

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}, \textcircled{\frac{1}{2}}$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \textcircled{\frac{1}{2}}$$

$$\text{SO } \sinh x = \tanh x \cosh x$$

$$= -\frac{2}{3} \cdot \frac{3}{\sqrt{5}}, \textcircled{1}$$

$$= -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}, \textcircled{\frac{1}{2}}$$

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $f(g(x))$.

SCORE: ____ / 5 PTS

You may need to use the exponential definition of cosh x.

$$\begin{aligned} & \cosh(\ln(x + \sqrt{x^2 - 1})) \\ &= \frac{e^{\ln(x + \sqrt{x^2 - 1})} + e^{-\ln(x + \sqrt{x^2 - 1})}}{2} \quad \textcircled{1} \\ &= \frac{x + \sqrt{x^2 - 1} + \frac{1}{x + \sqrt{x^2 - 1}}}{2} \cdot \frac{x + \sqrt{x^2 - 1}}{x + \sqrt{x^2 - 1}} \quad \textcircled{1} \\ &= \frac{(x + \sqrt{x^2 - 1})^2 + 1}{2(x + \sqrt{x^2 - 1})} \end{aligned}$$

$$\begin{aligned} &= \frac{x^2 + 2x\sqrt{x^2 - 1} + x^2 - 1 + 1}{2(x + \sqrt{x^2 - 1})} \quad \textcircled{1} \\ &= \frac{2x^2 + 2x\sqrt{x^2 - 1}}{2(x + \sqrt{x^2 - 1})} \\ &= \frac{2x(x + \sqrt{x^2 - 1})}{2(x + \sqrt{x^2 - 1})} \quad \textcircled{1} \\ &= x \quad \textcircled{1} \end{aligned}$$